INTERNATIONAL MATHEMATICS
TOURNAMENT OF TOWNS

SENIOR PAPER: YEARS 11,12

Tournament 39, Northern Spring 2018 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. An angle bisector and an altitude emanating from the same vertex of a triangle divide the opposite side into three parts. Is it possible that a new triangle may be constructed from those three parts? (3 points)

2. Four positive integers are given such that each of them is divisible by the greatest common divisor of the other three numbers, and the least common multiple of any three is divisible by the fourth number. Prove that the product of these four numbers is a perfect square. (4 points)

3. Two circles Γ_1 and Γ_2, with centres O_1 and O_2 respectively, touch externally at point T. A common tangent touches Γ_1 at point A and Γ_2 at point B. A common tangent to both circles at point T meets the line AB at point M. Suppose AC is a diameter of Γ_1. Prove that CM and AO_2 are perpendicular to each other. (4 points)

4. There is a checker in the corner square of an 8×8 chessboard. Petya and Vasya take turns moving the checker. Petya starts first, and on his turn he moves as a chess queen, where only the final square that the checker is moved over is considered *used*. Vasya on his turn makes a double move as a chess king, where both squares moved over are considered *used*. The checker cannot be moved over a *used* square. The initial square is also considered *used*. The player who cannot make a move loses. Who of the boys can play so that he will win for sure, no matter how his opponent moves? (5 points)

5. A convex polyhedron is given with exactly three faces meeting at each vertex. Each face of the polyhedron is coloured red, yellow or blue. The vertices, where the faces of all three colours meet, are called *multicoloured*. Prove that the number of multicoloured vertices is even. (5 points)